ALLAMA IQBAL OPEN UNIVERSITY, ISLAMABAD
(Department of Mathematics & Statistics)

WARNING

1. **PLAGIARISM OR HIRING OF GHOST WRITER(S) FOR SOLVING THE ASSIGNMENT(S) WILL DEBAR THE STUDENT FROM AWARD OF DEGREE/CERTIFICATE, IF FOUND AT ANY STAGE.**
2. **SUBMITTING ASSIGNMENTS BORROWED OR STOLEN FROM OTHER(S) AS ONE'S OWN WILL BE PENALIZED AS DEFINED IN "AIOU PLAGIARISM POLICY".**

Course: Mathematics-I (1307)

Level: F.A/F.Sc **Semester: Autumn, 2013**
Total Marks: 100 **Pass Marks: 40**

ASSIGNMENT No. 1
(Units 1–5)

Note: Attempt all questions, each question carry equal marks

Q.1 a) Simplify: i. $(-ai)^4, a \in R$ ii. i^{-31} (6)

b) Prove that $\sqrt{5}$ is an irrational number. (4)

c) Define complex numbers and separate the following into real and imaginary parts. (10)

(i) $\frac{i}{1+i}$ (ii) $\frac{(-2+3i)^2}{(1+i)}$

Q.2 Solve the following systems of homogeneous linear equations. (8)

(i) $x + 2y - 2z = 0$
$2x + y + 5z = 0$
$5x + 4y + 8z = 0$

(ii) $x_1 - 2x_2 - x_3 = 0$
$x_1 + x_2 + 5x_3 = 0$
$2x_1 - x_2 + 4x_3 = 0$

(b) If

$A = \begin{bmatrix} 1 \\ 1+i \\ i \end{bmatrix}$, find $A(A)^t$

(c) Solve the following system of linear equations by Cramer's rule. (6)

$$2x_1 - x_2 + x_3 = 8$$
$$x_1 + 2x_2 + 2x_3 = 6$$
$$x_1 - 2x_2 - x_3 = 1$$

Q.3 (a) Show that the set $\{1, \omega, \omega^2\}$, when $\omega^3 = 1$, is an Abelian group w.r.t ordinary multiplication. (10)

(b) Prove that: $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$ (10)

Q.4 (a) If α, β are the roots of the equation $ax^2 + bx + c = 0$, form the equation whose roots are (12)

(i) α^3, β^3 (ii) $-\frac{1}{\alpha^3}, -\frac{1}{\beta^3}$ (iii) $\alpha + \frac{1}{\alpha}$, $\beta + \frac{1}{\beta}$

(b) If ω is a cube root of unity, form an equation whose roots are 2ω and $2\omega^2$. (8)

Q.5 (a) If A = $\begin{bmatrix} i & 1+i \\ 1 & -i \end{bmatrix}$, show that

(i) A + $(\overline{A})^t$ is hermitian (ii) A - $(\overline{A})^t$ is skew hermitian (12)

(b) Show that the roots of $(mx + c)^2 = 4ax$ will be equal, if $c = \frac{a}{m}; m \neq 0$ (8)

ASSIGNMENT No. 2

(Units 6–9)

Note: Attempt all questions, each question carry equal marks

Q.1 (a) Resolve the following into partial fractions: (12)

(i) $\frac{2x-5}{(x^2+2)^2(x-2)}$

(ii) $\frac{4x^2+3x^3+6x^2+5x}{(x-1)(x^2+x+1)^2}$

(b) If $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in A.P. , Show that $b = \frac{2ac}{a+c}$ (8)

Q.2 (a) If a^2, b^2 and c^2 are in A.P., show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P. (6)

(b) Find three consecutive numbers in G.P whose sum is 26 and their product is 216. (6)

(c) For what value of n, $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the positive geometric mean between a and b? (8)

Q.3 (a) If the numbers $\frac{1}{k}, \frac{1}{2k+1}$ and $\frac{1}{4k-1}$ are in harmonic sequence, find k? (10)

(b) Find the sum to infinity of the series; $r + (1+k)r^2 + (1+k+k^2)r^3 + \cdots$ r and k being proper fractions. (10)

Q.4 (a) Two coins are tossed twice each. Find the probability that the head appears on the first toss and the same faces appear in the two tosses. (6)

(b) A natural number is chosen out of the first fifty natural numbers. What is the probability that the chosen numbers is a multiple of 3 or of 5? (6)

(c) Prove that ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$ (8)

Q.5 (a) Use mathematical induction to prove $2 + 6 + 18 + \cdots + 2 \times 3^{n-1} = 3^n - 1$ for every positive integer n. (6)

(b) Find 6th term in the expansion of $(x^2 - \frac{3}{2x})^{10}$. (6)

(c) If x is very nearly equal 1, then prove that $px^p - qx^q \approx (p-q)x^{p+q}$ (8)